Data Assimilation Introit

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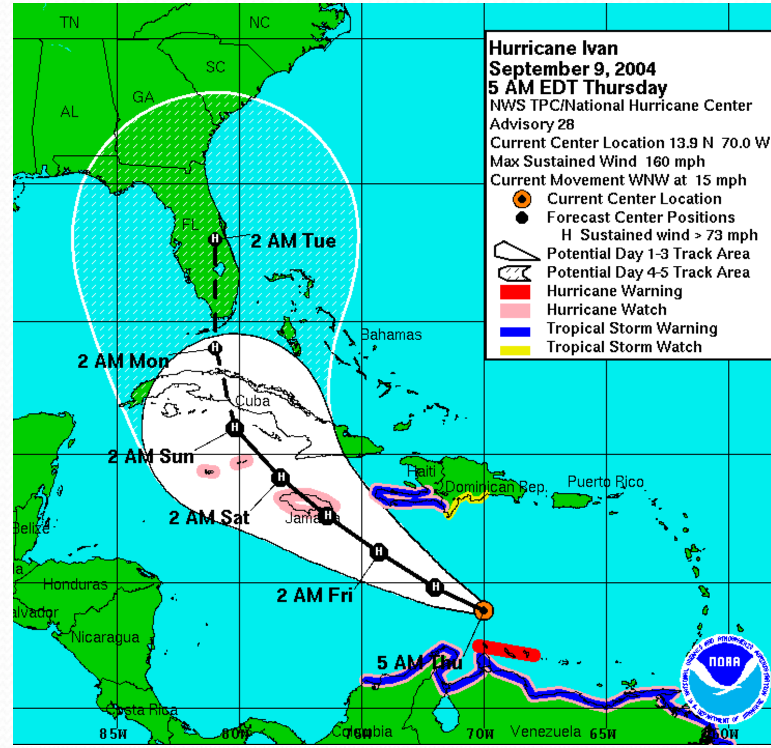
The next 90 minutes of your life

- Data Assimilation Introit
- Different methodologies
- Barnes Analysis in IDV

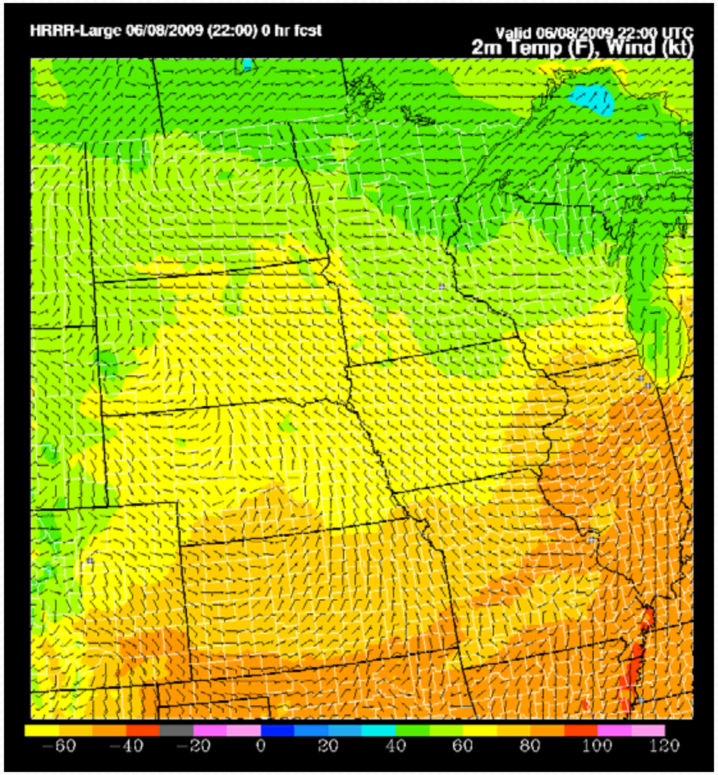
NWP Error Sources

1. Intrinsic Predictability Limitations
   a) Is the system inherently chaotic?
2. Errors in the Model
   a) Does the model represent the system correctly?
   b) Is model resolution sufficient?
   c) Are unresolved physical processes well parameterized?
3. Errors in the Initial Conditions and Boundary Conditions



Data Assimilation Introit

- Data assimilation is the procedure of getting the information content of observations into the numerical modeling system.
- The output of data assimilation is a maximum likelihood (optimal) estimate of the atmosphere, called an analysis, which can be used as the initial conditions for a numerical model.



Data Assimilation Introit

- Optimal analysis of the atmosphere obtained by combining a model forecast (first guess / background field) with observations.
- Some issues to attend to with data:
  - Data can be anything, and sometimes can be parameters that are not the fundamental state variables of the governing equations (radiance, for example)
  - Data can be of varying quality (instrument error, error of representation) – observations are not perfect
  - Data coverage may not be complete (500mb temperature over Cheyenne, WY, no sonde site) – need first guess

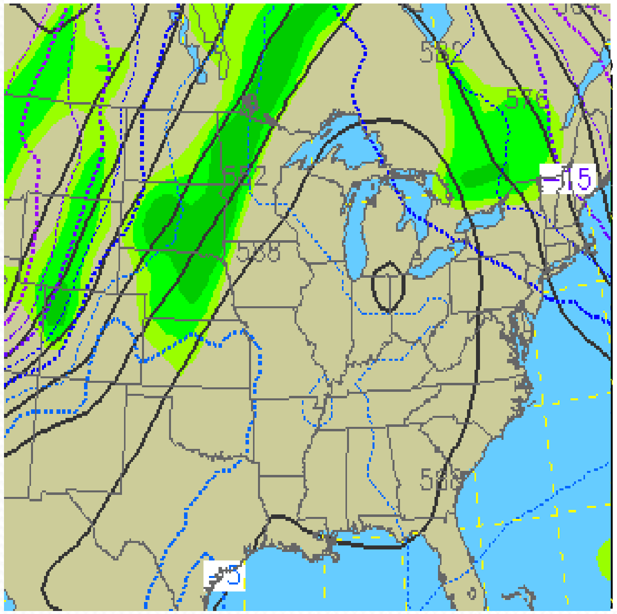
Data Assimilation Introit

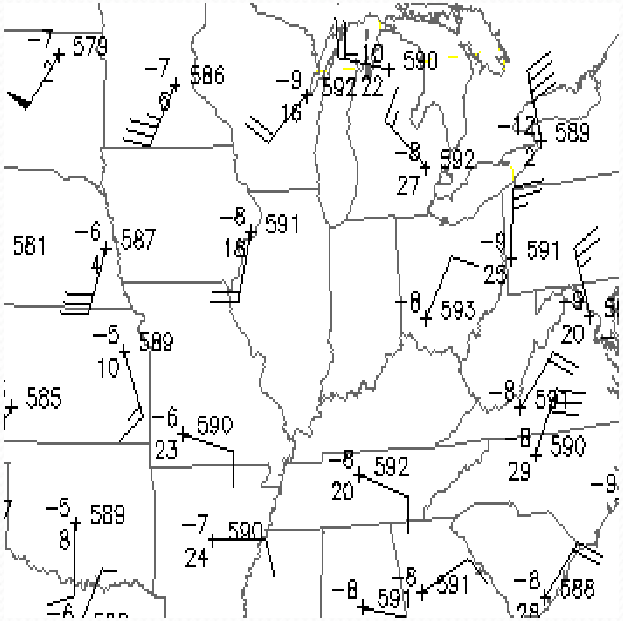
- Optimal analysis of the atmosphere obtained by combining a model forecast (first guess / background field) with observations.
- The first guess field / background field is usually a short-range forecast from the previous analysis cycle
  - The first guess at 1200 UTC could be the 6-hour forecast from the 0600 UTC model
- First guess field provides all state variables (T, p, q, etc.) at all grid point locations.
- First guess needs to be moved closer to observed values (it’s just a guess, after all).

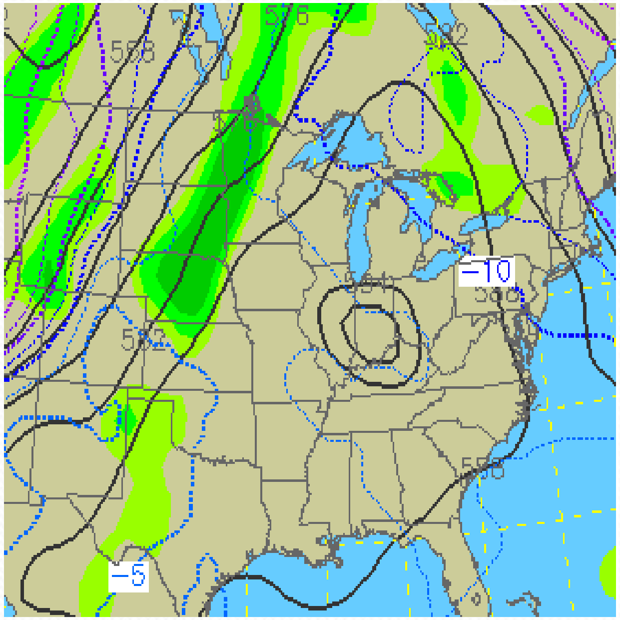
Data Assimilation Introit

- Optimal analysis of the atmosphere obtained by combining a model forecast (first guess / background field) with observations.
- Some issues with this:
  - Observational data is irregular in space and time
  - Model data is on a grid at regular times
  - Perfect fit to data – might get spurious results in model
  - Perfect fit to model – might be missing / under-representing important features
- Great to get the best of both the obs and first guess

Data Assimilation Introit







First Guess + Observations = Analysis

How does one optimally combine the first guess field and the observations?

Data Assimilation – One Value

- Consider two estimates of the temperature in this room.
  - T_F shall be what we set the thermostat to (a forecast)
  - T_O shall be the value from Mohan's watch (an observation)
- Use average squared errors (Variance) to weight the two estimates
  - Where σ_O^2 = Error Variance associated with T_O
  - Where σ_F^2 = Error Variance associated with T_F
- The optimal estimate (most likely value) of the temperature in the room, (T_A), is:

$$T_A - T_F = (\sigma_F^2)(\sigma_F^2+\sigma_O^2)^{-1}[T_O - T_F]$$

Data Assimilation – One Value

- The estimate of the temperature with minimum error variance, the analysis value ($\mathbf{T}_A$), is:

$$\mathbf{T}_A - \mathbf{T}_F = (\sigma_F^2)(\sigma_F^2 + \sigma_O^2)^{-1}[\mathbf{T}_O - \mathbf{T}_F]$$

- What if the thermostat is perfect?
  - Then $\sigma_F^2 = 0$
  - Then $\mathbf{T}_A - \mathbf{T}_F = 0$, so $\mathbf{T}_A = \mathbf{T}_F$
- What if Mohan's watch is perfect?
  - Then $\sigma_O^2 = 0$
  - Then $\mathbf{T}_A - \mathbf{T}_F = \mathbf{T}_O - \mathbf{T}_F$, so $\mathbf{T}_A = \mathbf{T}_O$
- $\mathbf{T}_A$ is a weighted average of the observation and first guess

Data Assimilation – Full model

$$T_A - T_F = (\sigma_F^2)(\sigma_F^2 + \sigma_O^2)^{-1}[T_O - T_F]$$

Assuming that observation and forecast errors are uncorrelated, the analysis increment ($\mathbf{x}^a$ - $\mathbf{x}^f$) that minimizes analysis error variance is (Cohn, 1997):

$$\underline{\mathbf{x}}^a - \underline{\mathbf{x}}^f = \underline{\mathbf{B}}\underline{\mathbf{H}}^T(\underline{\mathbf{H}}\underline{\mathbf{B}}\underline{\mathbf{H}}^T + \underline{\mathbf{R}})^{-1}[\underline{\mathbf{y}} - \mathbf{H}\underline{\mathbf{x}}^f]$$

Analysis equation

$$T_A - T_F = (\sigma_F^2)(\sigma_F^2+\sigma_O^2)^{-1}[T_O - T_F]$$

$$\underline{\mathbf{x}}^a - \underline{\mathbf{x}}^f = \underline{\mathbf{B}}\underline{\mathbf{H}}^T(\underline{\mathbf{H}}\underline{\mathbf{B}}\underline{\mathbf{H}}^T+\underline{\mathbf{R}})^{-1}[\underline{\mathbf{y}}-\underline{\mathbf{H}}\underline{\mathbf{x}}^f]$$

Model State Vector ($\underline{\mathbf{x}}^a$,$\underline{\mathbf{x}}^f$)

- A vector with all model variables at all model gridpoints

Observation Vector ($\underline{\mathbf{y}}$)

- A vector equal with all observations

Observation Operator ($\underline{\mathbf{H}}$)

- Converts data from observation space into model space (spatial conversion, variable conversion)

Analysis equation

$$T_A - T_F = (\sigma_F^2)(\sigma_F^2 + \sigma_O^2)^{-1}[T_O - T_F]$$

$$\underline{\mathbf{x}}^a - \underline{\mathbf{x}}^f = \underline{\mathbf{B}}\underline{\mathbf{H}}^T(\underline{\mathbf{H}}\underline{\mathbf{B}}\underline{\mathbf{H}}^T + \underline{\mathbf{R}})^{-1}[\underline{\mathbf{y}} - \underline{\mathbf{H}}\underline{\mathbf{x}}^f]$$

Analysis Increment ($\underline{\mathbf{x}}^a - \underline{\mathbf{x}}^f$)

- A vector equal to the difference between the analysis and the first guess field.

Innovation Vector ($\underline{\mathbf{y}} - \underline{\mathbf{H}}\underline{\mathbf{x}}^f$)

- A vector equal to the difference between the observations and the first guess field, in observation space

Analysis equation

$$T_A - T_F = (\sigma_F^2)(\sigma_F^2+\sigma_O^2)^{-1}[T_O - T_F]$$

$$\mathbf{x}^a - \mathbf{x}^f = \mathbf{BH}^T(\mathbf{HBH}^T+\mathbf{R})^{-1}[\mathbf{y}-\mathbf{Hx}^f]$$

Observation error covariance matrix (**R**)

- A matrix describing how errors of one observed parameter at one observing site correlate to errors of all observations (including itself)

First guess/Background error covariance matrix (**B**)

- A matrix describing how errors of one parameter at one gridpoint correlate to errors of all parameters at all gridpoints (including itself)

Analysis equation

$$\mathbf{x}^a - \mathbf{x}^f = \mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1}[\mathbf{y} - \mathbf{H}\mathbf{x}^f]$$

Depending how you make **B**, this equation can be used for ‘optimal interpolation’, or the ‘Kalman Filter’.

Data Assimilation Analogy: driving with your eyes closed: open eyes every analysis time and correct trajectory

Variational Schemes

$$T_A - T_F = (\sigma_F^2)(\sigma_F^2+\sigma_O^2)^{-1}[T_O - T_F]$$

Multiplying both sides by $(\sigma_F^2+\sigma_O^2)$, gives:

$$(\sigma_F^2+\sigma_O^2)[T_A - T_F] = (\sigma_F^2)[T_O - T_F]$$

Expand out left hand side:

$$[T_A - T_F](\sigma_F^2)+[T_A - T_F](\sigma_O^2) = [T_O - T_F](\sigma_F^2)$$

Combining:

$$[T_A - T_F](\sigma_O^2) = [T_O - T_F](\sigma_F^2)-[T_A - T_F](\sigma_F^2)=[T_O - T_A](\sigma_F^2)$$

Divide all by $(\sigma_F^2\sigma_O^2)$:

$$[T_A - T_F](\sigma_F^2)^{-1}=[T_O - T_A](\sigma_O^2)^{-1}$$

Minimize a cost-function:

$$J(\underline{\mathbf{x}}^a) = (1/2)[(\underline{\mathbf{x}}^a-\underline{\mathbf{x}}^f)\underline{\mathbf{B}}^{-1}(\underline{\mathbf{x}}^a-\underline{\mathbf{x}}^f)]+(1/2)[(\underline{\mathbf{y}}-\underline{\mathbf{H}}\underline{\mathbf{x}}^a)\underline{\mathbf{R}}^{-1}(\underline{\mathbf{y}}-\underline{\mathbf{H}}\underline{\mathbf{x}}^a)]$$

Variational Schemes

Minimize a cost-function:

$$J(\underline{\mathbf{x}}^a) = (^1/_2)[(\underline{\mathbf{x}}^a - \underline{\mathbf{x}}^f)\underline{\mathbf{B}}^{-1}(\underline{\mathbf{x}}^a - \underline{\mathbf{x}}^f)] + (^1/_2)[(\underline{\mathbf{y}} - \underline{\mathbf{H}}\underline{\mathbf{x}}^a)\underline{\mathbf{R}}^{-1}(\underline{\mathbf{y}} - \underline{\mathbf{H}}\underline{\mathbf{x}}^a)]$$

As with Optimal Interpolation, and the Kalman Filter, the challenge in data assimilation is how to estimate **B** and **R**, as well as proper construction of **H** .

R usually a diagonal matrix, as all observation errors are assumed to be uncorrelated. An exception is nadir-sounding satellites, with vertical observation error correlations

Variational Schemes

Minimize a cost-function:

$$J(\mathbf{x}^a) = (^1/_2)[(\mathbf{x}^a-\mathbf{x}^f)\mathbf{B}^{-1}(\mathbf{x}^a-\mathbf{x}^f)]+(^1/_2)[(\mathbf{y}-\mathbf{H}\mathbf{x}^a)\mathbf{R}^{-1}(\mathbf{y}-\mathbf{H}\mathbf{x}^a)]$$

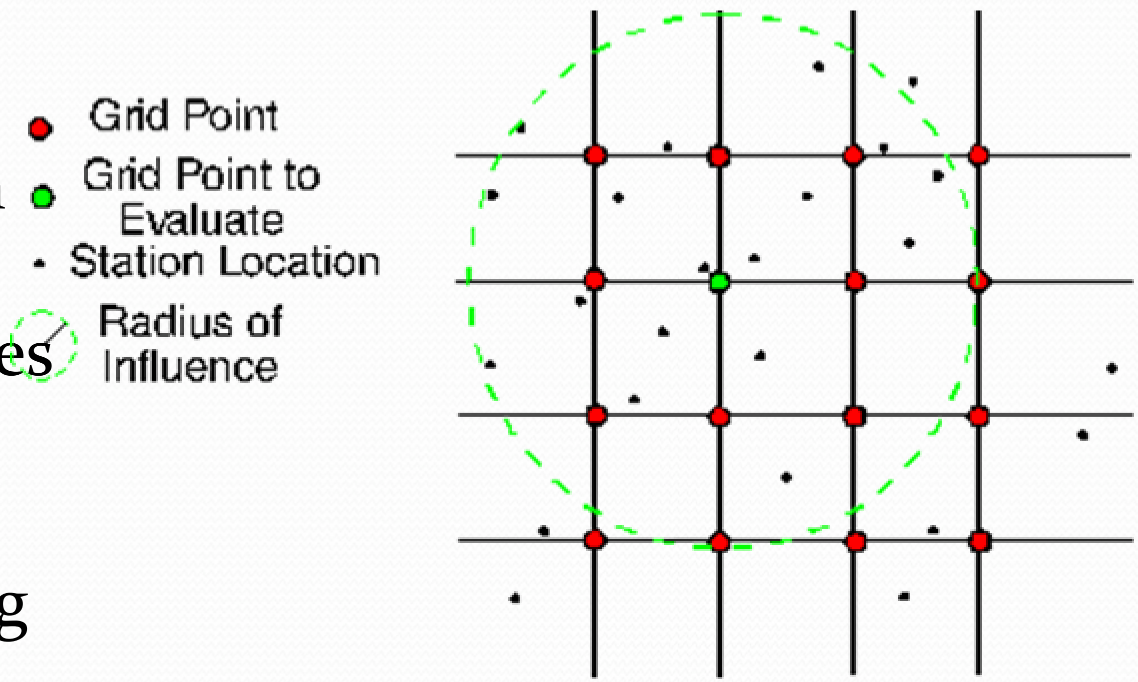
As with Optimal Interpolation, and the Kalman Filter, the challenge in data assimilation is how to estimate **B** and **R**

B can take many forms. It can be created by:

- Declaring a Gaussian shape (OI)
- Using a recent climatology (3D-Var)
- Using differences in forecasts valid at analysis time (EnKF)

Barnes Analysis

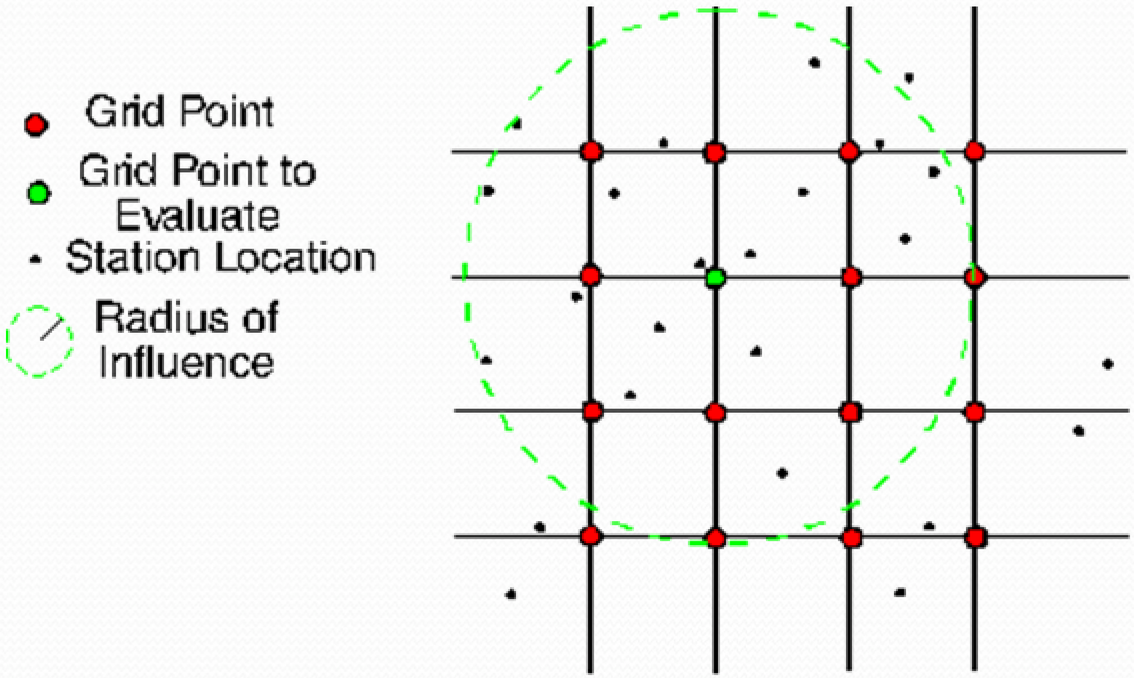
- The Barnes Scheme applies a Gaussian weighting function, in which the weight an observation contributes to the overall value of the grid point falls off rapidly with increasing distance from the point.



http://www.unidata.ucar.edu/software/gempak/tutorial/barnes.html

Barnes Analysis

- Since the tails of a Gaussian function are infinite, in practice a **radius of influence** is is chosen such that stations outside the circle about the gridpoint are not considered.



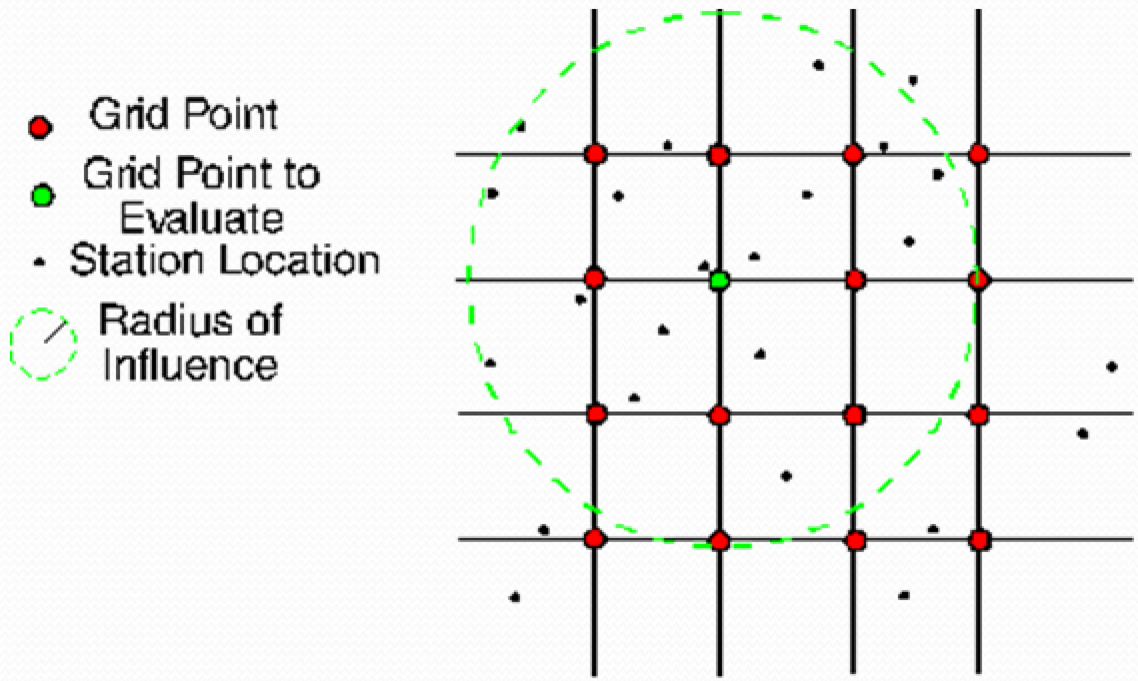
http://www.unidata.ucar.edu/software/gempak/tutorial/barnes.html

Barnes Analysis

- For each gridpoint, stations within the radius of influence are assigned a **weight value** “W” using the formula:

$$W_I = e^{-(d/R)^2}$$

where d is the distance from the observation to the gridpoint and R is the radius of influence.



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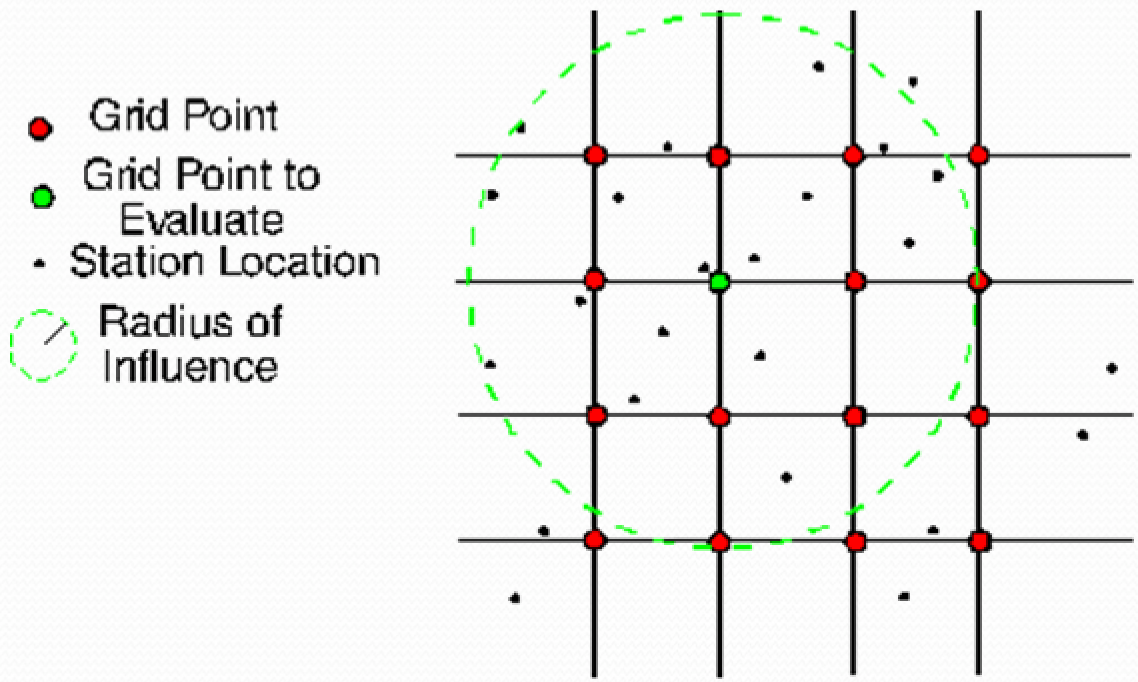
Barnes Analysis

- After the weights are determined the analysis value at the gridpoint is determined by:

$$X_g = \frac{\Sigma W_i X_i}{\Sigma W_i}$$

This is known as the first pass of the scheme



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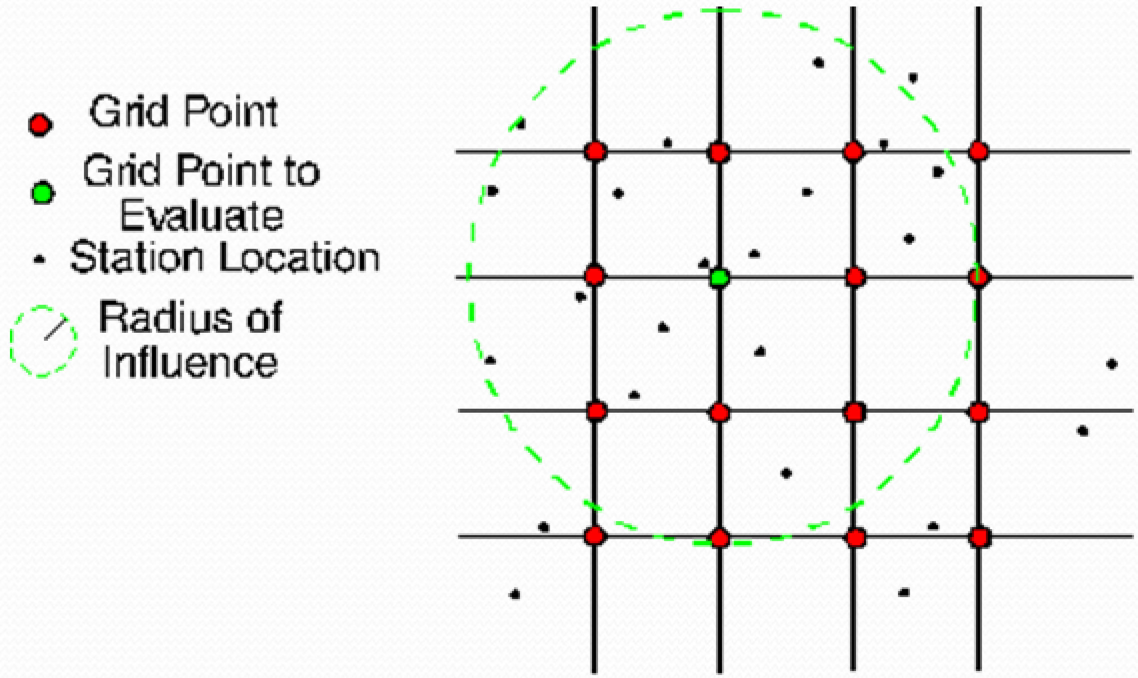
Barnes Analysis

- If more than 1 iteration of the scheme is desired (typically 2 **passes** are preformed), a method known as successive correction is applied.
- Each correction step can be represented as:

$$X'_g = X_g + \frac{\Sigma W'_i (X_i - X_a)}{\Sigma W'_i}$$



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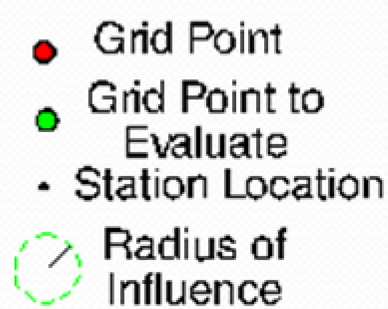
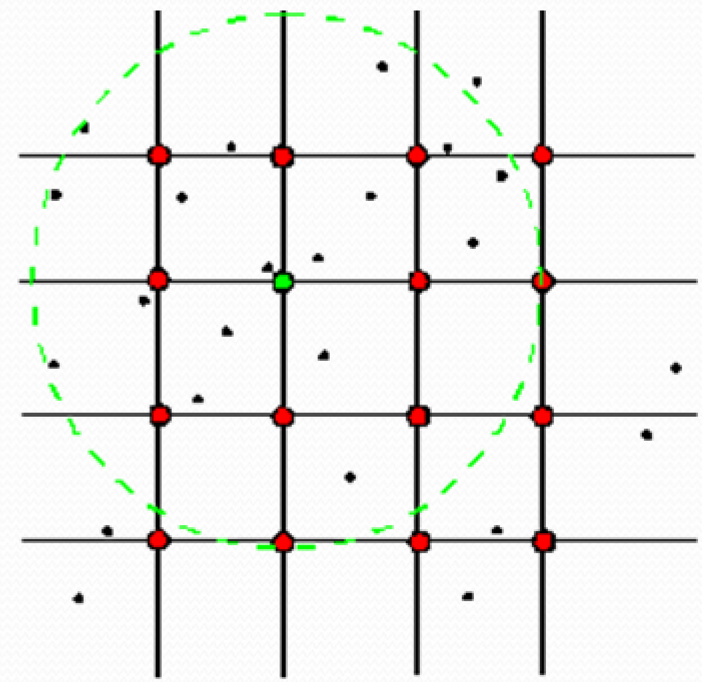
Barnes Analysis

- A new parameter called the **convergence parameter** (GAMMA) is use to control the amount of smoothing.



$$W_i = e^{-(d/R)^2} \quad W'_i = e^{-(d/R\Gamma)^2}$$

where d is the distance from the observation to the gridpoint and R is the radius of influence



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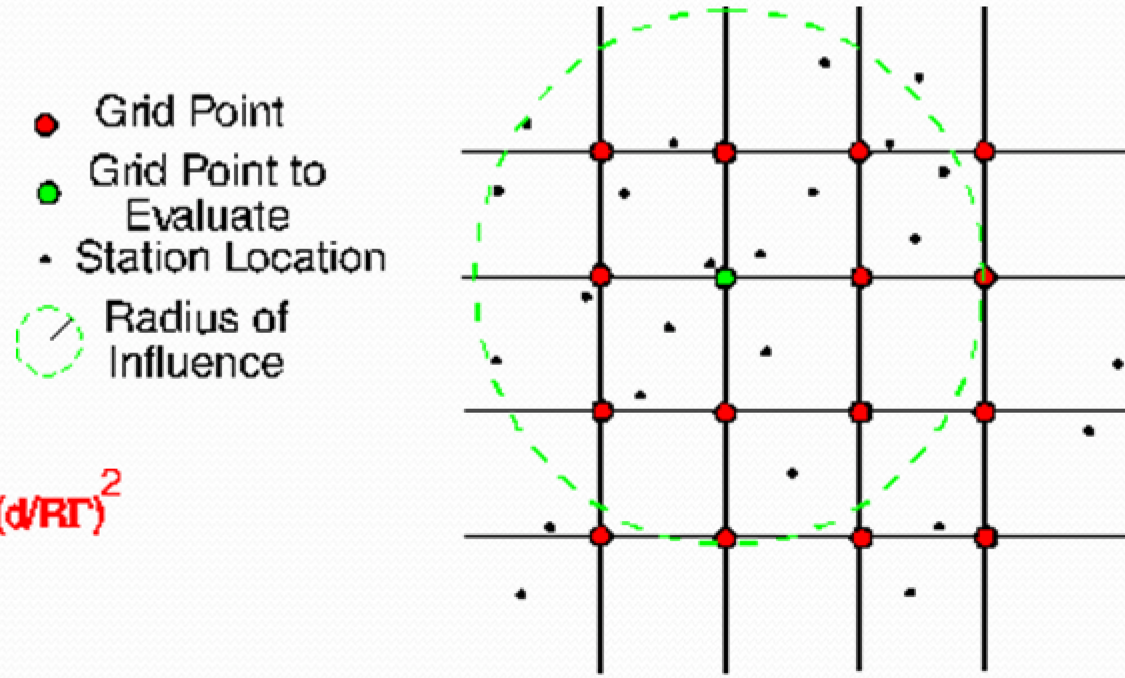
Barnes Analysis

- The convergence parameter gamma ranges between 0 and 1.
- A value between .2 and .3 is generally assumed.



$$W_i = e^{-(d/R)^2} \quad W'_i = e^{-(d/R\Gamma)^2}$$

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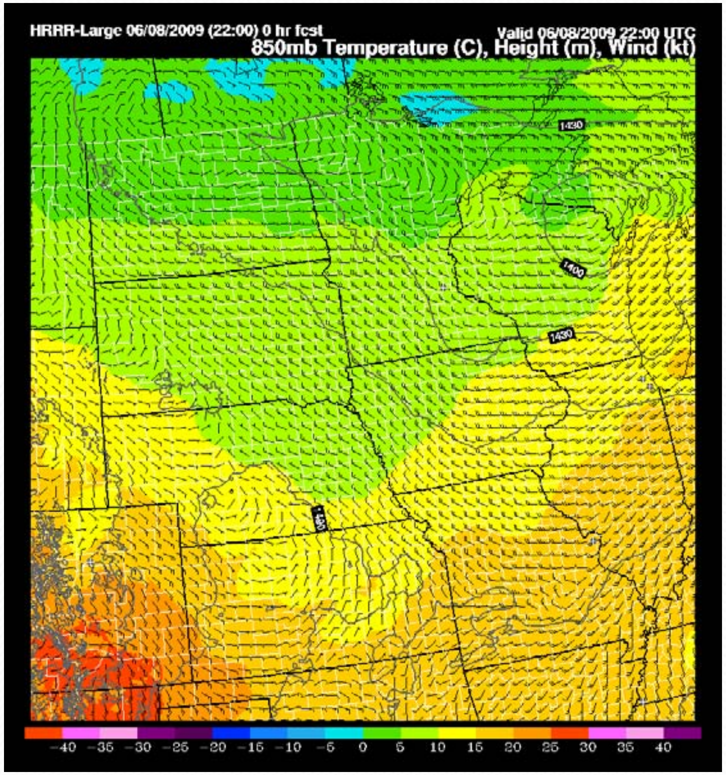
Barnes Analysis in IDV

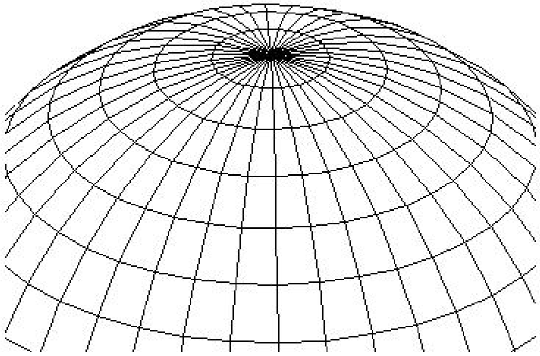
- The resulting grid can be displayed using any of the Gridded Data Displays.
- There are 5 parameters for you to set:
  - Spacing
  - Grid Size
  - Passes
  - Search Radius
  - Gain



http://mailman.unidata.ucar.edu/software/idv/docs/userguide/controls/ObjectiveAnalysis.html

Barnes Analysis in IDV

- **Spacing.** You can set the grid spacing as follows:
  - Automatic - grid spacing will be calculated from the observation density
  - Degrees - use a specific lat/lon spacing
  - Points - set the number of grid points in the x and y direction
- **Grid Size**. Specify the grid spacing if not using automatic calculation.



http://mailman.unidata.ucar.edu/software/idv/docs/userguide/controls/ObjectiveAnalysis.html

Barnes Analysis in IDV

- **Passes.** Set the number of passes for the Barnes analysis to do
  - 4 passes recommended for analyzing fields where derivative estimates are important (Ref: Barnes 1994b)
  - 3 passes recommended for all other fields (with gain set to 1.0) (Ref: Barnes 1994c "Two pass Barnes Objective Analysis schemes now in use probably should be replaced by appropriately tuned 3pass or 4pass schemes")
  - 2 passes only recommended for "quick look" type analyses.
  - 1 pass is rather rough
  - 5 is right out (Book of Armaments)

http://mailman.unidata.ucar.edu/software/idv/docs/userguide/controls/ObjectiveAnalysis.html

Barnes Analysis in IDV

- **Search Radius.** Set the search radius (in grid units) for the weighting of points in determining the value at a specific grid point. Should be in the range 0.2 to 1.0.
  - Data are fitted more closely with a gain of 0.2 (at the expense of less overall accuracy over the entire grid); larger values smooth more.
- **Gain.** Set the factor by which scale Length is reduced for the second pass. Should be in the range 0.2 to 1.0.
  - Data are fitted more closely with a gain of 0.2 (at the expense of less overall accuracy over the entire grid); larger values smooth more.
  - Suggested default: 1.0 Set the gain for each pass after the first.

http://mailman.unidata.ucar.edu/software/idv/docs/userguide/controls/ObjectiveAnalysis.html

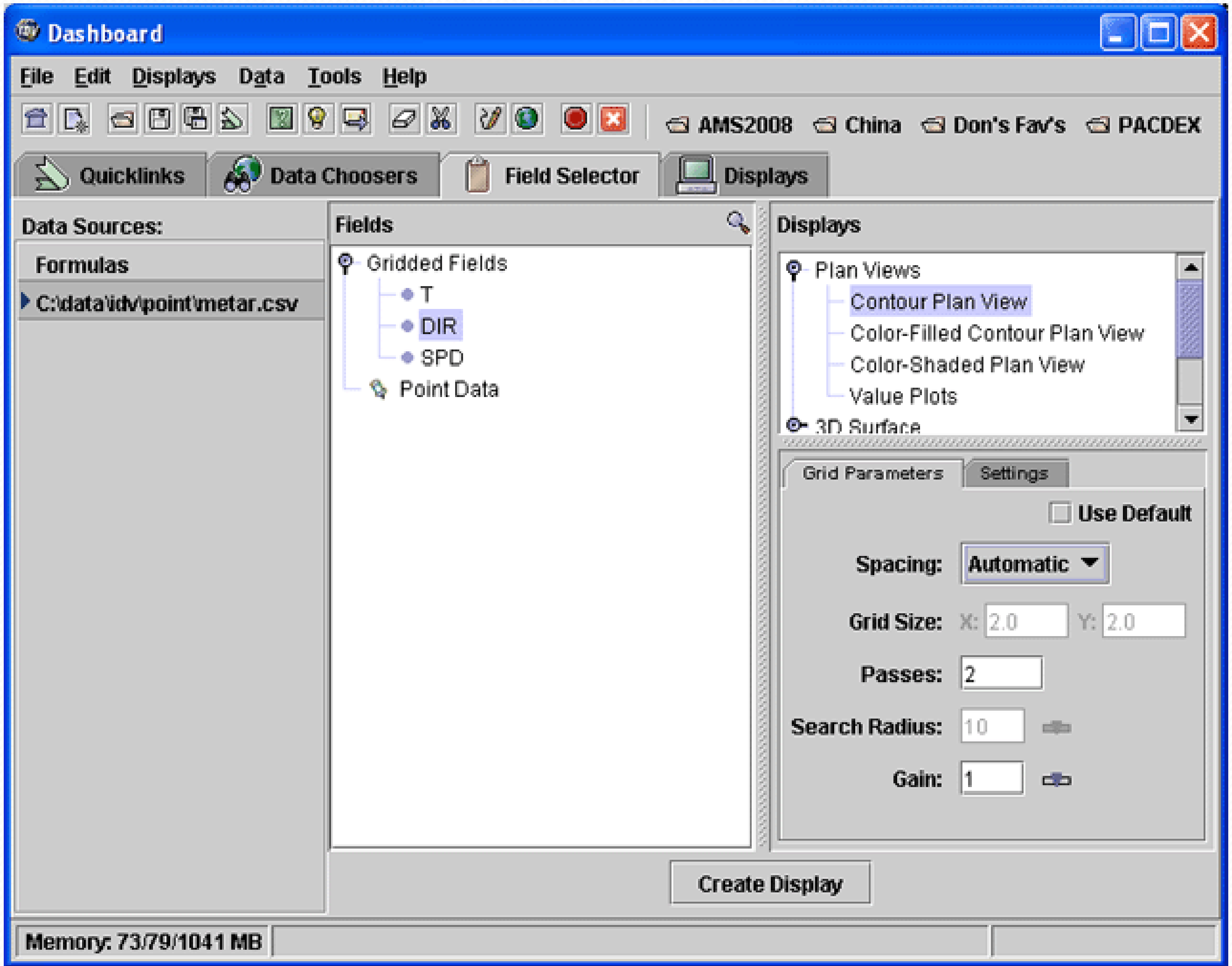
Now it is your turn

- Launch IDV!
- Load in sonde data
- Load in surface data
- Make some analyses

Load in the sonde and METAR data

- <u>Data</u>
  - Choose Data
    - Upper Air Observations
      - Connect
        - Most Recent
          - "Add Source"
- <u>Data</u>
  - Choose Data
    - Surface Observations
      - Connect
        - Most Recent
          - "Add Source"

Barnes Analysis in IDV



Default Settings:

- Spacing
  - 2 degrees
- Passes
  - 2
- Search Radius
  - 10
- Gain
  - 1
- Field
  - Pressure
- Level
  - Surface

Explore these areas

- See what happens if you change the grid-spacing from the default 2-degrees to 0.5-degrees or 10-degrees
- See what happens if you change the number of passes from 2 to 1 or 10
- See what happens if you change the search radius from 10 to 1
- See what happens if you change the gain from 1 to 0.1
- See what happens if you look at different fields (T, T_d)
- See what happens if you look at surface (METAR) data as opposed to 500mb data
- **Upload your favorite analyses to share with us**

Barnes Analysis in IDV

- References Barnes, S.L., 1994a: Applications of the Barnes objective analysis scheme Part I: Effects of undersampling, wave position, and station randomness. J. Atmos. Oceanic Technol. 11, 1433-1448.
- Barnes, S.L., 1994b: Applications of the Barnes objective analysis scheme Part II: Improving derivative estimates. J. Atmos. Oceanic Technol. 11, 1449-1458.
- Barnes, S.L., 1994c: Applications of the Barnes objective analysis scheme Part III: Tuning for minimum error. J. Atmos. Oceanic Technol. 11, 1459-1479.
- http://www.asp.ucar.edu/colloquium/1992/notes/part1/node120.html